

NPS55-88-002

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



### AN OPERATIONAL ANALYSIS OF SYSTEM CALIBRATION

DONALD P. GAVER

HASAN B. MUTLU

FEBRUARY 1988

Approved for public release; distribution is unlimited.

Prepared for:  
Office of Naval Research  
Arlington, VA 22217

FedDocs  
D 208.14/2  
NPS-55-88-002

NAVAL POSTGRADUATE SCHOOL  
MONTEREY, CALIFORNIA

Rear Admiral R. C. Austin  
Superintendent

K. T. Marshall  
Acting Provost

The work reported herein was supported in part with funds provided by the Office of Naval Research.

Reproduction of all or part of this report is authorized.

This report was prepared by:

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>			1b RESTRICTIVE MARKINGS		
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b DECLASSIFICATION / DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) <b>NPS55-88-002</b>			5 MONITORING ORGANIZATION REPORT NUMBER(S)		
5a NAME OF PERFORMING ORGANIZATION <b>Naval Postgraduate School</b>		6b OFFICE SYMBOL (If applicable) <b>Code 55</b>		7a NAME OF MONITORING ORGANIZATION	
5c ADDRESS (City, State, and ZIP Code) <b>Monterey, CA 93943-5000</b>			7b ADDRESS (City, State, and ZIP Code)		
8a NAME OF FUNDING / SPONSORING ORGANIZATION <b>Office of Naval Research</b>		8b OFFICE SYMBOL (If applicable) <b>Code 430</b>		9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER <b>N0001487WR24005</b>	
8c ADDRESS (City, State, and ZIP Code) <b>Arlington, VA 22217</b>			10 SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO <b>61153N</b>	PROJECT NO <b>RR014-05-01</b>	TASK NO <b>4114531-02</b>
			WORK UNIT ACCESSION NO		
11 TITLE (Include Security Classification) <b>AN OPERATIONAL ANALYSIS OF SYSTEM CALIBRATION</b>					
12 PERSONAL AUTHOR(S) <b>Gaver, Donald P. and Mutlu, Hasan B. (AT&amp;T Bell Laboratories)</b>					
13a TYPE OF REPORT <b>Technical</b>		13b TIME COVERED FROM _____ TO _____		14 DATE OF REPORT (Year, Month, Day) <b>1988 February</b>	
15 PAGE COUNT <b>20</b>					
16 SUPPLEMENTARY NOTATION					
17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	calibration, operational effectiveness, quality control, renewal theory, availability		
19 ABSTRACT (Continue on reverse if necessary and identify by block number)					
<p>Mathematical models are proposed for studying the impact of mis-calibration upon operational effectiveness. Methodology for assessing the system effectiveness and an approach for optimizing the effectiveness of a calibration program are examined. The theory and application are discussed, and the results of some specific and convenient models are presented.</p>					
20 DISTRIBUTION / AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21 ABSTRACT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>		
22a NAME OF RESPONSIBLE INDIVIDUAL <b>Donald P. Gaver</b>			22b TELEPHONE (Include Area Code) <b>(408)646-2605</b>		22c OFFICE SYMBOL <b>Code 55Gv</b>



## **An Operational Analysis of System Calibration**

*Donald P. Gaver*

*Naval Postgraduate School*

*Hasan B. Mutlu*

*AT&T Bell Laboratories*

Mathematical models are proposed for studying the impact of mis-calibration upon operational effectiveness. Methodology for assessing the system effectiveness and an approach for optimizing the effectiveness of a calibration program are examined. The theory application is discussed and the results of some specific and convenient models are presented.

### *1. Introduction*

The effectiveness of many systems depends upon the degree of the calibration of their subsystems. For example, a ship with navigational equipment that is out of calibration may not be able to locate its destination, or, in the case of a Navy ship, locate an adversary. If a Navy ship's weapon system is also out of calibration the difficulties will be compounded. An analogous problem arises in connection with engine de-tuning, when fuel consumption likely increases and performance decreases, and with drift of communication systems. Similar reasoning also applies to hardware and/or software configurations of computer systems, e.g. in factory robotic systems where mechanical re-calibrations occur and are controlled by software systems. The detrimental effect of mis-calibration is well recognized; the systems are taken to ranges or other facilities for testing and recalibration.

The purpose of this paper is to develop simple mathematical models to investigate ways of dealing with mis-calibration. If the various aspects of the problem can be assembled, some guidance is then available to deal with it effectively. Although many realistic elements of the problem can be considered, the fundamental issue is this; given that important subsystems depart from calibration and effectiveness as time passes, it is desirable to determine a schedule for re-calibration to optimize system operational effectiveness. Frequent calibration of important systems would be highly desirable if this were a cost-free operation. However, in reality the operational cost of calibration is *time* during which the system is unavailable for, or so degraded as to be incapable of adequately performing, its operational purpose. It should be mentioned that the actual dollar cost in man-hours, of performing calibration and the degradation of the calibration equipment could be considered, but these aspects are ignored as secondary effects.

The problem discussed here is recognizably analogous to those formulated and analyzed in several other areas. Similar issues arise in industrial quality control; see Lorenzen and Vance (1986), who utilize renewal-reward theory, and Smith and Vemuganti (1968), who invoke standard Bayesian decision theory. The important area of system availability monitoring, in both military and civilian nuclear power industry, requires a similar analysis; Thomas, Jacobs and Gaver (1987) recently formulated such problems and solved them using dynamic programming techniques. But the present paper is intended to provide a skeletally simple model for a particular situation, leaving mathematical elaboration aside. Furthermore, we emphasize *operationally* relevant costs and penalties (i.e., times out of service), and not simple monetary costs, although such can be brought in.

Figure 1 is an idealized graph of operational effectiveness against time. The periods of duration  $C$  denote those periods during which the system has zero effectiveness because it is undergoing calibration and is therefore out of the operational area, and the periods of duration  $T$  represent those periods during which the system is operational, but diminishing in effectiveness. The graph suggests that, if effectiveness drops with time, there will be an optimal value for  $T$ , a *best* period,  $T^*$ , at which to calibrate. We now present how such a period may be determined. More complex and realistic models are also introduced.

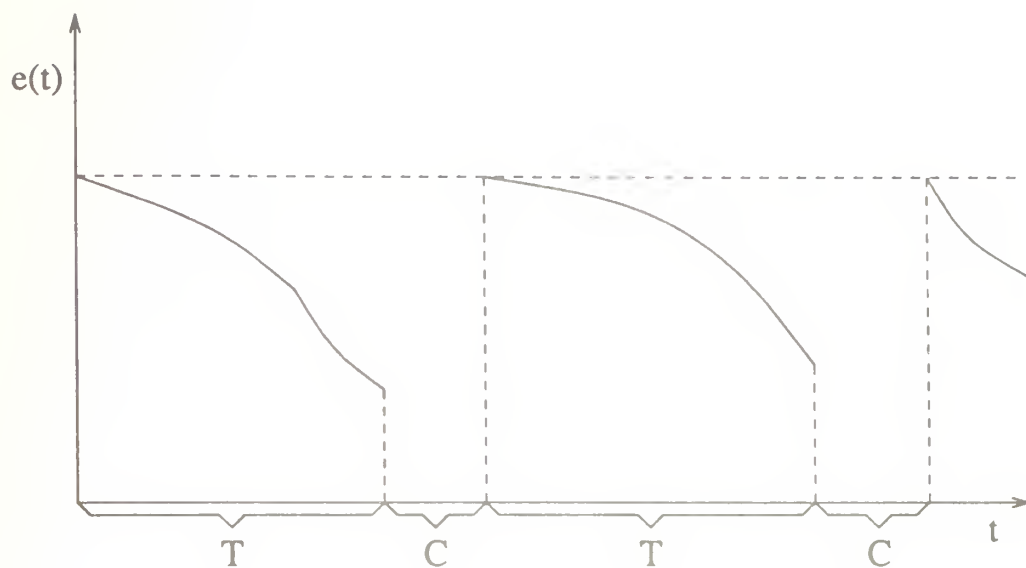


Figure 1. Idealized Graph of Operational Effectiveness.

## 2. Mathematical Models

For a simple mathematical treatment let  $e(t)$  be the effectiveness (e.g., the probability of successful operation), at time  $t$  after the calibrated system returns to service. Let  $C$  be the time required for calibration, and  $T$  the duty or on-station time. Then the average effectiveness over a cycle of length  $T + C$ , and hence in the long run, is

$$\bar{e}(T) = \frac{\int_0^T e(t)dt + 0}{T + C} ; \quad (2.1)$$

the term  $0$  represents and emphasizes the total lack of effectiveness during the calibration period. In order to maximize  $\bar{e}(T)$  it is useful to study the derivative



$$\frac{d\bar{e}(T)}{dT} = \frac{(T + C)e(T) - \int_0^T e(t)dt}{(T+C)^2} \quad (2.2)$$

as it depends on  $T$ : if  $d\bar{e}(T)/dT = 0$  for  $T^* > 0$  then  $T^*$  is a candidate for a time between the end of one calibration and the beginning of the next. Equivalently, equation (2.2) asks if there is a positive solution  $T^*$ , of

$$e(T) = \frac{1}{T+C} \int_0^T e(t)dt \quad (2.3)$$

for fixed positive  $C$ . The fact that such a solution always exists and that it defines an optimum can be established from the usual second derivative criterion. Since the optimal  $T$  satisfies equation (2.3), it turns out that at the optimum the average effectiveness over an entire cycle equals the effectiveness at the time the active part of the cycle ends; or symbolically as follows:

$$\bar{e}(T^*) = e(T^*) \quad (2.4)$$

where: the over-bar signifies the time average of effectiveness over  $T^* + C$ . All of the above tacitly assumes that the function  $e(t)$  is regular enough for the mathematical operations invoked:  $e(t) > 0$  decreasing and twice-differentiable is more than adequate; if  $e(t)$  is a probability then  $e(t) \leq 1$ .

To increase understanding, we will examine some simple specific models.



## 2.1 Linear Effectiveness Loss

Let

$$e(t) = \begin{cases} 1 - at, & 0 \leq t \leq a^{-1} \\ 0, & a^{-1} \leq t \end{cases} \quad (2.5)$$

so that the downward-sloping parts of the graph of Figure 1 are strictly linear. Then equation (2.3), the equation for optimal  $T = T^*$ , is as follows:

$$1 - aT = \frac{1}{T+C} \left( T - \frac{a}{2} T^2 \right), \quad 0 \leq T \leq a^{-1}. \quad (2.6)$$

It is clear that no value of  $T > a^{-1}$  can be optimum. The equation (2.6) simplifies to the quadratic as follows:

$$aT^2 + 2aCT - 2C = 0 \quad (2.7)$$

with a single positive solution

$$T^* = -C + \sqrt{C^2 + 2C/a} \quad (2.8)$$

at which the optimum value of effectiveness is as follows:

$$e(T^*) = \frac{\int_0^{T^*} e(t) dt}{T^* + C} = 1 - aT^*$$

$$\begin{aligned}
 &= (1 + aC) - \sqrt{a^2 C^2 + 2aC} \\
 &= (1 + aC) - \sqrt{(1 + aC)^2 - 1} .
 \end{aligned} \tag{2.9}$$

It is interesting that the solution depends only upon the parameter  $aC$ , the product of calibration drift rate,  $a$ , and the length of the re-calibration period,  $C$ . For instance, if  $aC \rightarrow 0$  then effectiveness approaches unity if the rate of calibration degradation,  $a$ , approaches zero, or the calibration time,  $C$ , approaches zero, or both, or one approaches zero more rapidly than the other gets large. Alternatively, this shows that equal-effectiveness or  $a$ - $C$  tradeoff curves are simple hyperbolas in the  $(a,C)$  plane. A tendency in this direction may well be common.

The above model is rather crude, but is easy to understand. It resembles the tool wear problem of Smith and Vemuganti (1968) without the Bayesian refinements, and with operationally relevant costs. The following model (Linear Degradation with Diffuse Damage) is more qualitatively appealing.

## 2.2 Linear Degradation with Diffuse Damage

Consider a more specific model for effectiveness, one that relates to damage inflicted on a target after time  $t$  has elapsed, and the system has developed an (unsuspected) bias of magnitude  $at$ . At this time, the  $x$ - $y$  error made in locating a target is assumed to be given by the joint Gauss/normal density as follows:

$$f(x,y;t) = \frac{1}{2\pi\sigma^2} \exp \left[ -\frac{1}{2} \frac{(x-at)^2}{\sigma^2} - \frac{1}{2} \frac{(y-at)^2}{\sigma^2} \right]. \tag{2.10}$$

If a cookie-cutter damage function, with radius  $R$ , is in effect (0% effective if  $x^2 + y^2 > R^2$ , 100% effective if  $x^2 + y^2 \leq R^2$ ) then:

$$e(t) = \iint_{x^2+y^2 \leq R^2} f(x,y;t) dx dy .$$

However, this is difficult to work with, and even overly simplistic. Instead, suppose that a von Neumann-Gauss diffuse damage function can be used; (i.e., that the probability of critical damage to a target located at  $(0,0)$  by a weapon with impact point  $(x,y)$  is equal to  $\delta(x,y) = \exp(-\alpha(x^2 + y^2))$  (Eckler and Burr (1972)). Then the following:

$$\begin{aligned}
 e(t) &= \iint \delta(x,y) f(x,y;t) dx dy \\
 &= \frac{1}{2\pi\sigma^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp[-\alpha(x^2+y^2)] f(x,y;t) dx dy \\
 &= \left( \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2} \frac{(at-x)^2}{\sigma^2}\right] \exp[-\alpha x^2] dx \right)^2
 \end{aligned} \tag{2.11}$$

by virtue of the symmetry assumed; almost free of charge we can consider asymmetrical damage functions, but the opportunity is declined. The above integral is evaluated at sight: it is seen to be essentially the convolution of two normal densities. After squaring, as demanded by equation (2.11), the result is:

$$e(t) = \frac{1/2\alpha}{(\sigma^2+1/2\alpha)} \exp\left[-\frac{(at)^2}{(\sigma^2+1/2\alpha)}\right]. \tag{2.12}$$

Instead of dropping off linearly, as in the previous case,  $e(t)$  first diminishes rather slowly, later falling quite rapidly (exponentially fast) towards zero: by the time  $at = \sqrt{\sigma^2 + 1/2\alpha}$ , effectiveness  $e(t)$  is just below 40% of its maximum, while if  $at = 0.5\sqrt{\sigma^2 + 1/2\alpha}$ , effectiveness  $e(t)$  is about 78% of the maximum; finally if  $at = 0.25\sqrt{\sigma^2 + 1/2\alpha}$ , effectiveness  $e(t)$  is 94% of the maximum. Note that the maximum effectiveness is  $(1 + 2\alpha\sigma^2)^{-1} \leq 1$ ; if either  $\sigma^2$  or  $\alpha$  become large, meaning that if either weapon effectiveness falls off rapidly, with miss distance ( $\alpha$  large), or the ultimate weapon delivery variance is great ( $\sigma^2$  large), then even maximum effectiveness is low.

In order to solve for the optimum  $T^*$  write (from equation (2.3)) the following:

$$(T+C)\exp\left[-\frac{(aT)^2}{(\sigma^2+1/2\alpha)}\right] = \int_0^T \exp\left[-\frac{(at)^2}{(\sigma^2+1/2\alpha)}\right] dt . \quad (2.13)$$

Change the variables to the dimensionless version as follows:

$$\tau = (aT)/(\sigma^2+1/2\alpha)^{1/2} ; \quad \gamma = (aC)/(\sigma^2+1/2\alpha)^{1/2} , \quad (2.14)$$

so one can solve the following dimensionless equation, once and for all, for  $\tau^*$ :

$$(\tau+\gamma)\exp(-\tau^2) = \int_0^\tau \exp(-z^2) dz ; \quad (2.15)$$

the positive value of  $\tau$ , namely  $\tau^*$ , that satisfies this equation may be located by Newton-Raphson, or even graphically: one can plot, for given  $\gamma$ ,

$$L(\tau) = (\tau+\gamma)\exp(-\tau^2)$$

and

$$R(\tau) = \int_0^\tau \exp(-z^2) dz$$

on the same piece of paper, vs.  $\tau$ .

The arbitrary selected  $\gamma$  values and the corresponding  $\tau^*$  values from the computer program, which solves the dimensionless equation (2.15), are shown in Table 1.

$\gamma$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$\tau^*$	0.236	0.302	0.346	0.381	0.410	0.434	0.456	0.476	0.494

TABLE 1.  $\gamma$  and  $\tau^*$  Values.

In order to redefine  $\tau$  and  $\gamma$  in equation (2.14) and the effectiveness formula equation (2.12) in a more meaningful form, again change variables to the following:

$$\nu = 1/2\alpha ; \quad p = \frac{\nu}{\sigma^2 + \nu} ; \quad k = a/\sigma \quad (2.16)$$

where:  $\nu$  and  $p$  might be called *vulnerability* and *probability of success* respectively, and  $k$  is constant. So one can write the following:

$$\tau = kT\sqrt{1-p} ; \quad \gamma = kC\sqrt{1-p} . \quad (2.17)$$

We focus attention on the representation equation (2.12) in what follows, mainly for analytical and computational convenience:

$$e(t) = p \exp[-(kt)^2(1-p)] . \quad (2.18)$$

Thus, the preceding expression, at the optimum point  $T^*$ , leads to the following relationship:

$$e(T^*) = p \exp(-(\tau^*)^2) \quad (2.19)$$

and consequently from equation (2.17), the optimal *proportion of on-station time* can be obtained as follows:

$$\frac{\tau^*}{\gamma} = \frac{T^*}{C}$$
$$T^* = \frac{\tau^*}{k\sqrt{1-p}} \quad , \quad (2.20)$$

so the proportion of on-station time is, under optimum conditions, as follows:

$$\frac{T^*}{T^*+C} = \frac{\tau^*}{\tau^*+\gamma} \quad (2.21)$$

Since the optimum  $\tau$  values are available from Table 1, one can easily calculate the optimal effectiveness, (2.4) (hereafter called simply effectiveness) given some constant variance and  $\nu$  or only  $p$ . Some of the results are tabulated in Table 2 as an example. Note that the effectiveness decreases as the variance increases while  $\nu$  is held constant.

**Example:** Suppose  $a = 1.5$  yds/month,  $C = 0.5$  month,  $\sigma^2 = 20$  (yds)<sup>2</sup> and  $p = 0.9$  are given. First find  $\gamma$  from equation (2.17) as 0.053, then look up the corresponding  $\tau^*$  value from Table 1, which is 0.417. From equation (2.20)  $T^*$  is 3.93 months, from equation (2.21) the proportion of on-station time is 88.7%, and from equation (2.19) an average effectiveness of 75.6% can be obtained.

Effectiveness								
$(\sigma^2=10; \nu=200)$	0.901	0.869	0.845	0.824	0.805	0.789	0.773	0.759
$(\sigma^2=20; \nu=200)$	0.860	0.830	0.806	0.786	0.768	0.753	0.738	0.725

TABLE 2. Effectiveness for Constant Variance ( $\sigma^2$ ) and  $\nu$ .

### 2.3 Linear Degradation with Diffuse Damage Using Random Drift

As an alternative model, that incorporates the possibly different drift rates of different individual systems, suppose that the drift,  $\underline{a}$ , is a random variable with an appropriate distribution function instead of a constant as in equation (2.12), namely, the effectiveness conditional on  $\underline{a}$  is as follows:

$$e(t; \underline{a}) = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha)} \exp \left[ -\frac{(\underline{a}t)^2}{(\sigma^2 + 1/2\alpha)} \right]. \quad (2.22)$$

Then the expected average, or unconditional effectiveness over a cycle of length  $T + C$ , in the long run, becomes the following:

$$E[\bar{e}(T; \underline{a})] = \frac{\int_0^T E[e(t; \underline{a})] dt}{T + C}. \quad (2.23)$$

In order to be specific, and also so that explicit mathematical results are obtained, let  $\underline{a}^2$  have a *gamma* distribution function with parameters  $\lambda$  and  $\beta$ , as follows.



$$f_{\underline{a}^2}(x;\lambda,\beta) = \frac{\lambda e^{-\lambda x}(\lambda x)^{\beta-1}}{\Gamma(\beta)} \quad \lambda, \beta > 0, \quad (2.24)$$

and use the following for fixed  $\underline{a}^2$ , (i.e., the square of the drift rate away from calibration),

$$E(\underline{a}^2) = \underline{a}^2 = \frac{\beta}{\lambda}; \quad Var(\underline{a}^2) = \frac{\beta}{\lambda^2} = \frac{\underline{a}^4}{\beta}. \quad (2.25)$$

Now use equation (2.22) and (2.23) to obtain the following:

$$E[e(t;\underline{a})] = \frac{1/2\alpha}{(\sigma^2+1/2\alpha)} \int_0^\infty \exp\left[-x\left(\frac{t^2}{\sigma^2+1/2\alpha}\right)\right] f_{\underline{a}^2}(x;\lambda,\beta) dx.$$

After substituting the gamma density function, it is easy to see that the result of the integration yields the following:

$$E[e(t;\underline{a})] = \frac{1/2\alpha}{(\sigma^2+1/2\alpha)} \left[ \frac{\lambda}{\lambda + \frac{t^2}{\sigma^2+1/2\alpha}} \right]^\beta, \quad (2.26)$$

or equivalently, in view of equation (2.25) the following:

$$E[e(t;\underline{a})] = \frac{1/2\alpha}{(\sigma^2+1/2\alpha)} \left[ \frac{1}{1 + \frac{(at)^2}{(\sigma^2+1/2\alpha)\beta}} \right]^\beta. \quad (2.27)$$

Various analytical properties, of the previously described model, will now be recorded. These

properties provide useful insights into the behavior of the effectiveness at time  $t$ .

1. If  $\beta \rightarrow \infty$ , equation (2.27) becomes the following:

$$E[e(t;\underline{a})] = \frac{1/2\alpha}{\sigma^2 + 1/2\alpha} \exp[-(at)^2] \quad (2.28)$$

which reflects the fact that if  $\beta$  increases, the variance of  $\underline{a}$  in the distribution of drift rate decreases towards zero, and the situation reduces to that of previous model.

2. If  $\beta \rightarrow 1$ , then the following:

$$E[e(t;\underline{a})] = \frac{1/2\alpha}{(\sigma^2 + 1/2\alpha + (at)^2)} \quad (2.29)$$

which is *larger than* the effectiveness in the equal-drift case.

In order to solve for the optimum  $T^*$ , write the following for the general case:

$$\frac{T+C}{\left[1 + \frac{(aT)^2}{(\sigma^2 + 1/2\alpha)\beta}\right]^\beta} = \int_0^T \frac{1}{\left[1 + \frac{(at)^2}{(\sigma^2 + 1/2\alpha)\beta}\right]^\beta} dt \quad (2.30)$$

Change the variables as follows:

$$\tau = (aT)/(\sigma^2 + 1/2\alpha)^{1/2} ; \quad \gamma = (aC)/(\sigma^2 + 1/2\alpha)^{1/2} \quad (2.31)$$

so that one can solve the dimensionless equation as follows:

$$\frac{\tau+\gamma}{\left[1+\frac{\tau^2}{\beta}\right]^\beta} = \int_0^\tau \frac{dz}{\left[1+\frac{z^2}{\beta}\right]^\beta}; \quad (2.32)$$

the positive value of  $\tau$ , namely  $\tau^*$ , that satisfies this equation for any constant  $\beta$  may be found by a computer program. In fact, one may get the solution for the special case  $\beta = 1$  by making use of *arctg* integration for the right-hand side as follows:

$$\gamma = (\arctg \tau) (1 + \tau^2) - \tau. \quad (2.33)$$

In general, the right-hand integral can be transformed to the integral of a *Student's t density*, and the *t-tables* found in most statistics books can be used to evaluate it.

The arbitrary selected  $\gamma$  values and the corresponding  $\tau^*$  values for  $\beta = 1$  are presented in Table 3.

$\gamma$	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$\tau^*$	0.239	0.307	0.355	0.392	0.424	0.451	0.476	0.499	0.520

TABLE 3.  $\gamma$  and  $\tau^*$  Values ( $\beta = 1$ ).

At this point, it is very easy to calculate the effectiveness given some constant variance and  $\nu$  or only  $p$  from equation (2.19). Some of the results are listed in Table 4 as an example. As in the previous case, effectiveness decreases as the variance increases when  $\nu$  is held constant.

Effectiveness								
$(\sigma^2=20; \nu=150)$	0.835	0.806	0.784	0.765	0.748	0.733	0.719	0.706
$(\sigma^2=30; \nu=150)$	0.788	0.761	0.740	0.722	0.706	0.692	0.679	0.667

TABLE 4. Effectiveness for Constant Variance ( $\sigma^2$ ) and  $\nu$  ( $\beta = 1$ ).

### 3. Conclusions

The purpose of this paper has been to show that quite simple mathematical models can provide useful ways of studying the impact of mis-calibration upon operational effectiveness. Methodology for assessing the system effectiveness and an approach for optimizing the effectiveness of a calibration program have been examined. We have concentrated here on specific and convenient models, but it is obvious that other more elaborate mathematical models can be treated similarly. The relative effectiveness of different system configurations can also be investigated.

The possibility exists that operational data will reveal different underlying distributions, and may suggest alternatives for evaluating effectiveness other than the ones described in this paper. This possibility could be profitably investigated.

REFERENCES

1. Lorenzen, T. J. and Vance, L. C. 1986. "The Economic Design of Control Charts: A Unified Approach." *Technometrics*, vol. 28, No. 1, pp. 3-10.
2. Smith, B. E. and Vemuganti, R. R. 1968. "A Learning Model for Processes with Tool Wear." *Technometrics*, vol. 10, No. 2, pp. 379-387.
3. Thomas, L. C., Jacobs, P. A., and Gaver, D. P. 1987. "Optimal Inspection Policies for Standby Systems." *Communications in Statistics: Stochastic Models*, vol. 3, No. 2, pp. 259-274.
4. Eckler, A. R. and Burr, S. A. 1972. *Mathematical Models of Target Coverage and Missile Allocation*. Military Operations Research Society (MORS).

DISTRIBUTION LIST

	<u>NO. OF COPIES</u>
Library (Code 0142) Naval Postgraduate School Monterey, CA 93943-5000	2
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Office of Research Administration (Code 012) Naval Postgraduate School Monterey, CA 93943-5000	1
Center for Naval Analyses 4401 Ford Avenue Alexandria, VA 22302-0268	1
Library (Code 55) Naval Postgraduate School Monterey, CA 93943-5000	1
Operations Research Center, Rm E40-164 Massachusetts Institute of Technology Attn: R. C. Larson and J. F. Shapiro Cambridge, MA 02139	1
Koh Peng Kong OA Branch, DSO Ministry of Defense Blk 29 Middlesex Road SINGAPORE 1024	1
Arthur P. Hurter, Jr. Professor and Chairman Dept of Industrial Engineering and Management Sciences Northwestern University Evanston, IL 60201-9990	1
Institute for Defense Analysis 1800 North Beauregard Alexandria, VA 22311	1
Chief of Naval Research 800 N. Quincy St. Arlington, VA 22217-5000	1







DUDLEY KNOX LIBRARY



3 2768 00331379 2